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Correction

Erratum to: Questionable Use of the Mathematical Concept of Equivalence by Psychologists

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Note

As the author conducted this work entirely privatly, without using any resources of the National Veterinary Assay Laboratory, the affiliation and contact information mentioned in the original article should be ignored. Accordingly, email must be addressed to author's personal address nihonsinrigakkai@gmail.com.

[The author requested to add this note post-publication on 2014-11-13.]

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Theoretical Analyses

Questionable Use of the Mathematical Concept of Equivalence by Psychologists

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Abstract

Psychologists have applied the mathematical concept of an equivalence relation to such topics as concept formation and foundations of language. This line of research is not without controversies, and most researchers have only intuitive understanding of this mathematical concept. In this article, accessible explanations are provided on fundamental issues that have implications for empirical research.

Keywords: equivalence, concept formation, foundations of language

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Psychologists have borrowed the mathematical concept of an equivalence relation and applied it to empirical investigations on issues such as concept formation and foundations of language. This line of research is not without controversies (mentioned later), and most researchers have only intuitive understanding of this mathematical concept. In this brief article, accessible explanations are provided on fundamental issues. Researchers are strongly encouraged to review their research framework.

Briefly, in mathematics, an equivalence relation (R) is defined by the following three axioms:

a R a (reflexivity)

if **a R b** then **b R a** (symmetry)

if **a R b** and **b R c** then **a R c** (transitivity)

Researchers have tried to apply this to their experiments on animals or language-impaired children (e.g., see Sidman, 2009, for an introductory review). For a typical experiment, the experimenter presents a sample stimulus, and the subject chooses among two or more stimuli, only one of which is designated correct. If the subject chooses the correct answer, s/he will be told to be correct, or the animal will be rewarded with some food. For instance, when presented with the printed word “cat”, the child must choose the picture of a cat, not that of a pencil. On other trials, s/he may be presented with the picture of a book and s/he must verbally say “book”, not “flower”. In the case of animal studies, all the stimuli may be some abstract visual images and the animal may touch keys.

The training design is to present paring of A→B (e.g., a set of [printed word]→[picture] pairs) and B→C (e.g., a set of [picture]→[verbal response] pairs). The subject is then tested with *untrained* pairs B→A and C→B. So if the subject is presented with the picture of a cat and correctly chooses the printed word "cat", despite having been never explicitly trained in that pair, the experimenter concludes that the equivalence relation was established. Using this framework, researchers have often had difficulties in establishing symmetry, thus falling short of establishing the equivalence.

However, unfortunately, such a framework is not based on strong foundations and most psychologists' understanding of those foundations is not rigorous but only intuitive. Most researchers do not know any other mathematical relations. For instance, there is a partial order relation, where reflexivity and symmetry also hold. (Another axiom *antisymmetry* is required instead of symmetry for a partial order). As an easy example, the familiar inequality using ≤ is a partial order. (Note that $a \le a$ is correct, and if $a \le b$ and $b \le c$, then $a \le c$).

So why can we assume that the experimental subject knows the experimenter is testing an equivalence relation, not other relations such as a partial order? Particularly, the animal subject will not know the experimenter's intention and may learn in accordance with a partial order, which is perfectly consistent with the training. In that case, its performance in tests will only show reflexivity and transitivity but not symmetry. The author is not claiming that the experimental subjects learn a partial order; rather, the author warns that assuming learning of equivalence using the usual procedure is not warranted. Lack of knowledge on mathematical relations other than equivalence also might have led to the misunderstanding such as "Reflexivity means identity of the same object". Such a statement is only true in the context of an equivalence relation. Reflexivity in itself does not have such a meaning.

In addition, most psychologists may not know why reflexivity is needed, and this has implications on the experimental procedure. One of the questions asked by mathematics and engineering students taking mathematical courses is "Why is reflexivity necessary, as it can be derived from the other two (symmetry and transitivity)?" That is, many erroneously think that "as **a R b** implies **b R a** (symmetry), using transitivity, '**a R b** and **b R a**, therefore **a R a**' which is nothing other than reflexivity".

To understand why this is incorrect, one should remember that "if then" (material implication; p→q) used in logic is different from everyday usage of the language "if then". The truth table of material implication is shown in Table 1.

Table 1

Truth Table of Material Implication (cf. Magnus, 2005, p. 26)

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

Particularly notable is that the proposition is always true when p is false. This can lead to a seemingly absurd statement being correct such as "If 4 is an odd number, 1 is larger than 100". (Some introductory textbooks may somehow try to convince readers on this by giving verbal rationalization. But it will be more appropriate to say that material implication is simply *different* from everyday usage of "if then").

Now suppose we consider a relation “has an opposite sign”, which is clearly not an equivalence relation. So **+2 R -3** is true but **+5 R +1** is false. However, now let us restrict the set and reduce it from all numbers to limited numbers. So if we consider the set (+1, +2, +3), then the relation *never* holds. As a result, symmetry and transitivity hold for this relation because p is always false and thus the proposition is always true. But reflexivity does not hold. Therefore, without reflexivity, this relation is inappropriately concluded to be an equivalence relation. This is the reason why reflexivity is indispensable.

However, consider the usual procedure used in behavioral experiments. Two stimuli are always presented and shown to be related. So in these cases reflexivity is indeed derived from the other two (symmetry and transitivity). Therefore, reflexivity is not needed in behavioral experiments.

In fact, this framework of stimulus equivalence is not accepted by all psychologists as the basis of concepts or language, whether in animal or human research. A cursory look at psychological textbooks will show that many completely different frameworks are investigated in other areas of psychology (e.g., fuzzy logic). Also important to note is that already some researchers expressed doubts on the validity of this framework. The author sympathizes with Tonneau (2001) and Burgos (2003) who stated that the equivalence relation or the set theory in general are concepts of pure mathematics and should have no place in empirical sciences. In any case, it is preferable that researchers correctly understand basic mathematical concepts and then reconsider or advance equivalence research.

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Competing Interests

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